



Measurement of the Atmospheric Primary Aberrations by 4-Apertures *HARTMANN*

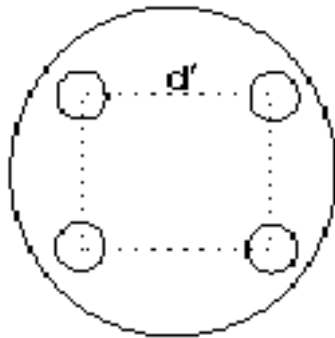
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Hartmann Test Using a Screen With Four Holes



<p>(a) <i>Astigmatism at 0° or 90°</i></p>	<p>(b) <i>Astigmatism at angle $\pm 45^\circ$</i></p>
<p>(c) <i>Astigmatism at angle θ</i></p>	<p>(d) <i>Coma in y direction</i></p>

Hartmann test with four holes screen

-The theoretical concept of the primary aberrations measurement by four apertures Hartmann test introduced by

“*Malacara*” * and “*Salas-Peimbert et. al.*” **

* *Malacara D, Malacara Z Opt. Eng. 31. 1551-55(1992).*

** *Salas-Peimbert et. al. Appl. Opt. 44, 4228-38(2005).*

Theory

Let us assume at Hartmann screen, each aperture is located on the corners of a square with side d' . The aberrations of the distorted wave-front at four apertures screen are defined by:

$$W(x, y) = Bx + Cy + D(x^2 + y^2) + E(x^2 - y^2) + Fxy + G(x^2 + y^2 - d'^2)y + H(x^2 + y^2 - d'^2)x,$$

B is tilt coefficient about the y axis,

C is tilt coefficient about the x axis,

D is the defocusing coefficient,

E is the astigmatism coefficient with the axis at 0 or 90,

F is the astigmatism coefficient with the axis at 45,

G is the Coma coefficient about the y axis,

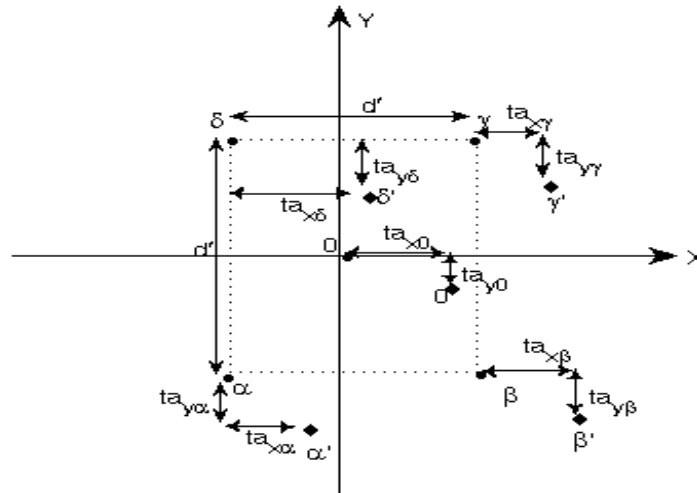
H is the Coma coefficient about the x axis.

The x and y components of the transverse aberrations are given by:

$$\frac{\partial W(x, y)}{\partial x} = \frac{ta_x}{F_{tel}} = B + 2Dx + 2Ex + Fy + 2Gxy + H(3x^2 + y^2 - d^2),$$

$$\frac{\partial W(x, y)}{\partial y} = \frac{ta_y}{F_{tel}} = c + 2Dy - 2Ey + Fx + G(x^2 + 3y^2 - d^2) + 2Hxy,$$

Where F_{tel} is the four apertures screen distance from CCD



We can determine seven aberration coefficients with eight transverse aberrations equations for all apertures as:

tilts: $B = \frac{ta_{y\alpha} + ta_{y\beta} + ta_{y\gamma} + ta_{y\delta}}{4F_{tel}}$, $C = \frac{ta_{x\alpha} + ta_{x\beta} + ta_{x\gamma} + ta_{x\delta}}{4F_{tel}}$,

Defocus: $D = - \frac{(ta_{x\alpha} - ta_{x\beta}) - (ta_{x\gamma} - ta_{x\delta}) + (ta_{y\alpha} + ta_{y\beta}) - (ta_{y\gamma} + ta_{y\delta})}{8F_{tel}d'}$.

Astigmatism:

$$E = - \frac{(ta_{x\alpha} - ta_{x\beta}) - (ta_{x\gamma} - ta_{x\delta}) - (ta_{y\alpha} + ta_{y\beta}) + (ta_{y\gamma} + ta_{y\delta})}{8F_{tel}d'}$$

$$F = - \frac{(ta_{x\alpha} + ta_{x\beta}) - (ta_{x\gamma} + ta_{x\delta}) + (ta_{y\alpha} - ta_{y\beta}) - (ta_{y\gamma} - ta_{y\delta})}{4F_{tel}d'}$$

Coma: $H = - \frac{(ta_{y\alpha} - ta_{y\beta}) + (ta_{y\gamma} - ta_{y\delta})}{2F_{tel}d'^2}$, $G = - \frac{(ta_{x\alpha} - ta_{x\beta}) + (ta_{x\gamma} - ta_{x\delta})}{2F_{tel}d'^2}$.

By this definition for the wave-front aberrations the centroid of these four spots (the average coordinates of the four spots) is not shifted by ***defocusing***, ***astigmatism*** and ***coma*** terms and it is shifted only by the two ***tilts***,

-The configuration of the system of four spots depends on the coefficients *D, E, F, G, H*

-The global position depends on the coefficients B & C.

“If there are no local tilts, the centroid of the four spots is located exactly in the position of ideal four spots centroid, so if we locate ideal four spots centroid in four spots centroid position and reconstruct ideal four spot by measured distance for each ideal spots, we could ignore tilt terms.”

Simulation

To test the ability of four apertures Hartmann for the measurement of atmospheric primary aberrations, We perform a numerical simulation. Here we explain:

1-How We Simulate of four spots images at the telescope focal plane

2-The result of Primary aberrations measurement by the four apertures Hartmann

Simulation of four spots images at the telescope focal plane

Let us suppose that a single star light has a perturbed phase, φ , and a uniform illumination, A , in front of the telescope aperture. We know from Fourier optics that the complex wave function on telescope aperture is

$$U_i = A \exp(-i\varphi).$$

For a telescope with pupil function P

$$P(x, y) = \begin{cases} 1 & \text{in_side_the_aperture} \\ 0 & \text{otherwise} \end{cases},$$

The complex wave function and the intensity distribution at the focal plane of the telescope are [18]:

$$U_f = FFT(P.U_i) = FFT(PA \exp(-i\varphi)),$$

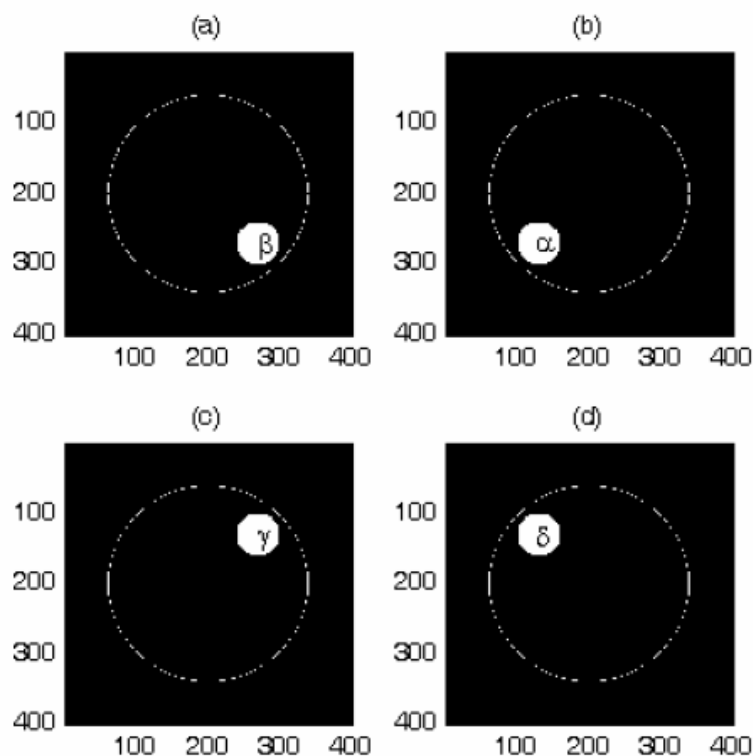
And

$$I_f = |U_f|^2,$$

respectively, where FFT stands for Fast Fourier transform. The simulation arrangement consists of a monochromatic light ($\lambda = 0.5 \mu m$) impinging on a telescope aperture with focal length $F_{tel} = 2.8m$ and aperture diameter $D_{tel} = 28cm$.

In aperture plane, we simulate a circular pupil with diameter 280 pixels, centered into a sampled rectangular matrix with 400×400 pixel resolution and $1\text{mm} \times 1\text{mm}$ pixel size. The observation plane is placed at the focal plane of telescope with 400×400 pixel resolution and $3.5\mu\text{m} \times 3.5\mu\text{m}$ pixel size.

For simulation of spots images in four apertures DIMM, as illustrated in figure (3), we use the following pupil functions for each aperture with radius $R = 3\text{cm}$.



$$P_{\alpha}(x, y) = \begin{cases} 1 & (x + 70)^2 + (y + 70)^2 \leq R^2 \\ 0 & \text{otherwise} \end{cases},$$

$$P_{\beta}(x, y) = \begin{cases} 1 & (x - 70)^2 + (y + 70)^2 \leq R^2 \\ 0 & \text{otherwise} \end{cases},$$

$$P_{\gamma}(x, y) = \begin{cases} 1 & (x - 70)^2 + (y - 70)^2 \leq R^2 \\ 0 & \text{otherwise} \end{cases},$$

$$P_{\delta}(x, y) = \begin{cases} 1 & (x + 70)^2 + (y - 70)^2 \leq R^2 \\ 0 & \text{otherwise} \end{cases}.$$

As pointed before and by use of the spots pupil functions, the image functions of spots could be calculated as follow:

$$I_{\beta} = \left| \text{FFT}(P_{\beta} \cdot \exp(-i(\varphi + Q(-z_2 - z_3)))) \right|^2,$$

$$I_{\alpha} = \left| \text{FFT}(P_{\alpha} \cdot \exp(-i(\varphi + Q(+z_2 - z_3)))) \right|^2,$$

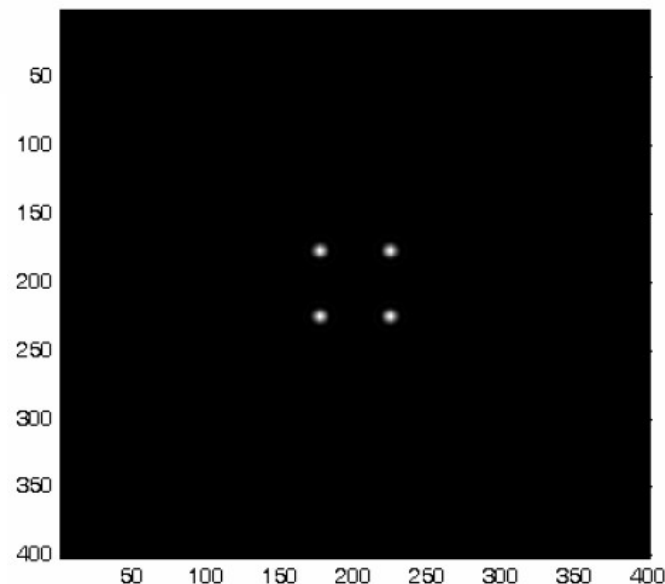
$$I_{\gamma} = \left| \text{FFT}(P_{\gamma} \cdot \exp(-i(\varphi + Q(-z_2 + z_3)))) \right|^2,$$

$$I_{\delta} = \left| \text{FFT}(P_{\delta} \cdot \exp(-i(\varphi + Q(+z_2 + z_3)))) \right|^2,$$

where z_2 and z_3 are the Zernike tilt terms which like a wedge separate each spot images and Q is the tilt coefficient. In our simulation we use Harding et. al. MATLAB source code to simulate a phase screen with Kolmogorov statistics using interpolative methods, which produces a Kolmogorov phase screen in the desired size[19]. The resultant image of four spots could be calculated by (Fig.4):

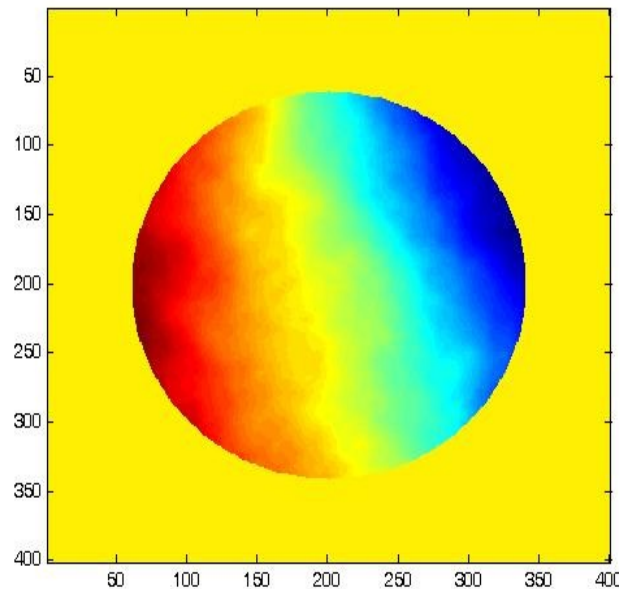
$$I = I_{\alpha} + I_{\beta} + I_{\gamma} + I_{\delta}.$$

Image of simulated four spots.

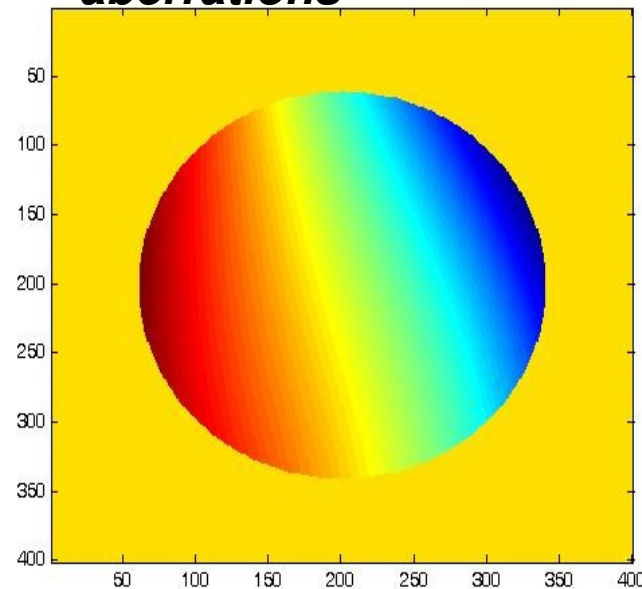


We generated set of atmospheric phase screen by Harding et. al. code's*. with 200 samples. To study the effect of the higher order aberrations, we decompose the generated phase screens into the Zernike modes generate the new set of phase screens. new phase screen includes 8 modes of Zernike aberrations. In fact in this new set, *we cut high order aberrations from original sets*. Thus, one may compare the calculated coefficients obtained by the original phase screen with those of the new phase screen with 8 Zernike modes. Using the introduced method we simulate four spots images and we calculate each spots centroid.

***generated phase screen
by Harding et. al. code's***

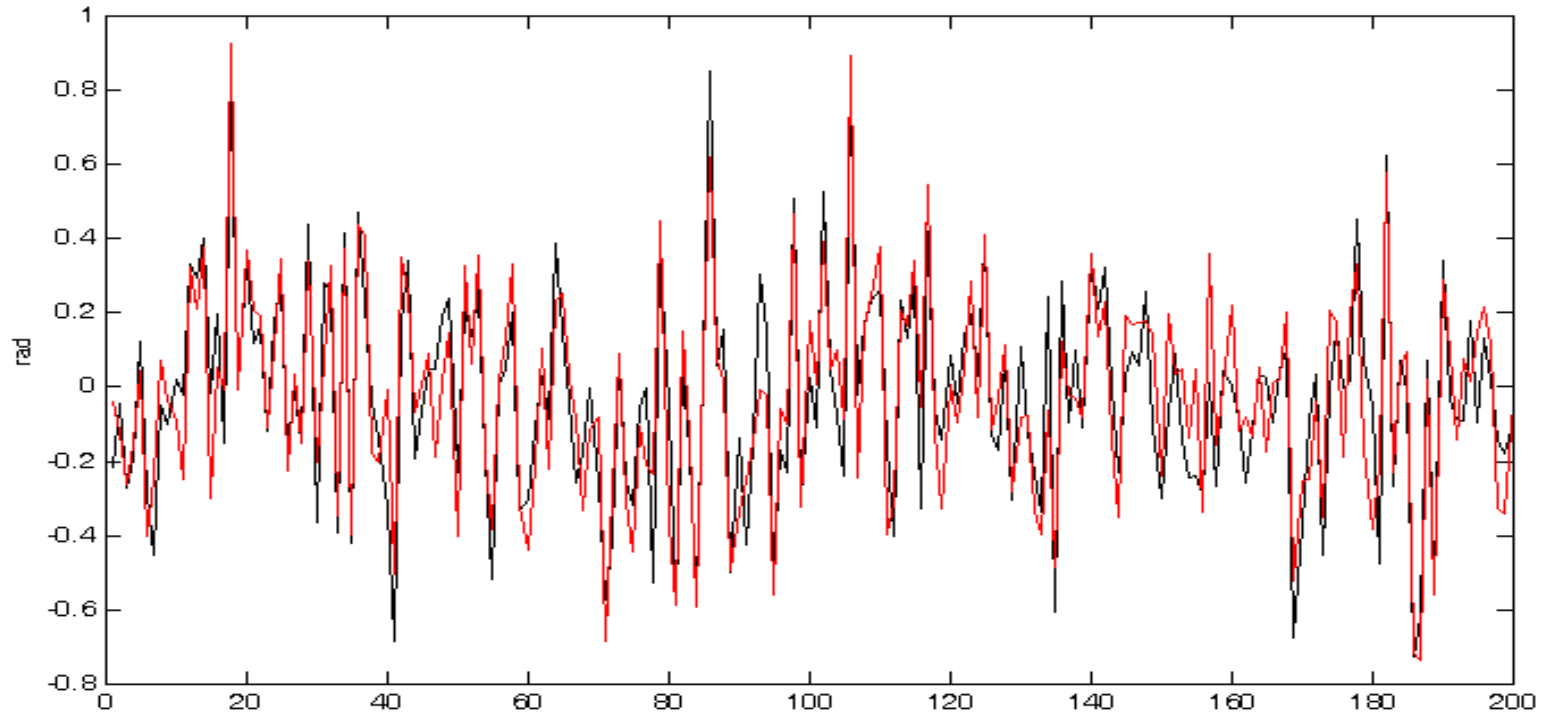


***phase screen includes 8
modes of Zernike
aberrations***



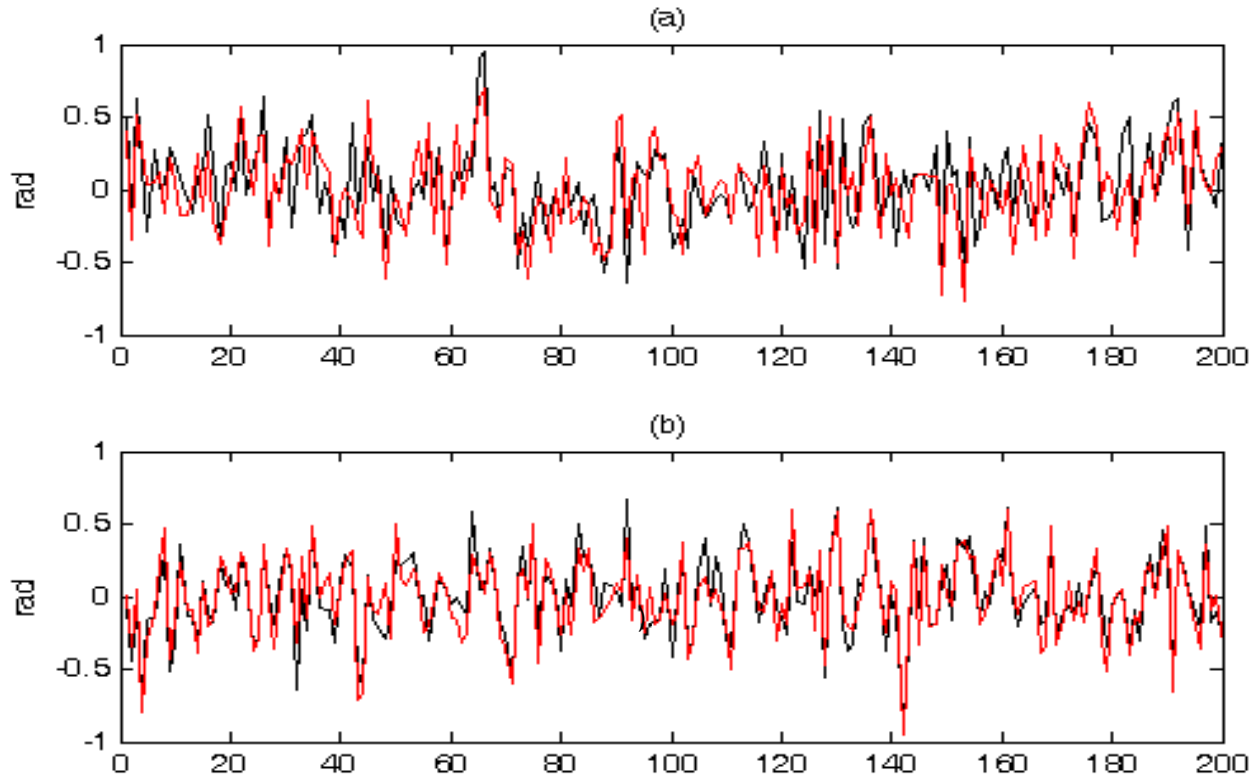
****Harding C M, Johnston R. A, Lane R G 1999 "Fast Simulation of a Kolmogorov Phase Screen" Appl. Opt. 38, 2161-70.***

Defocus



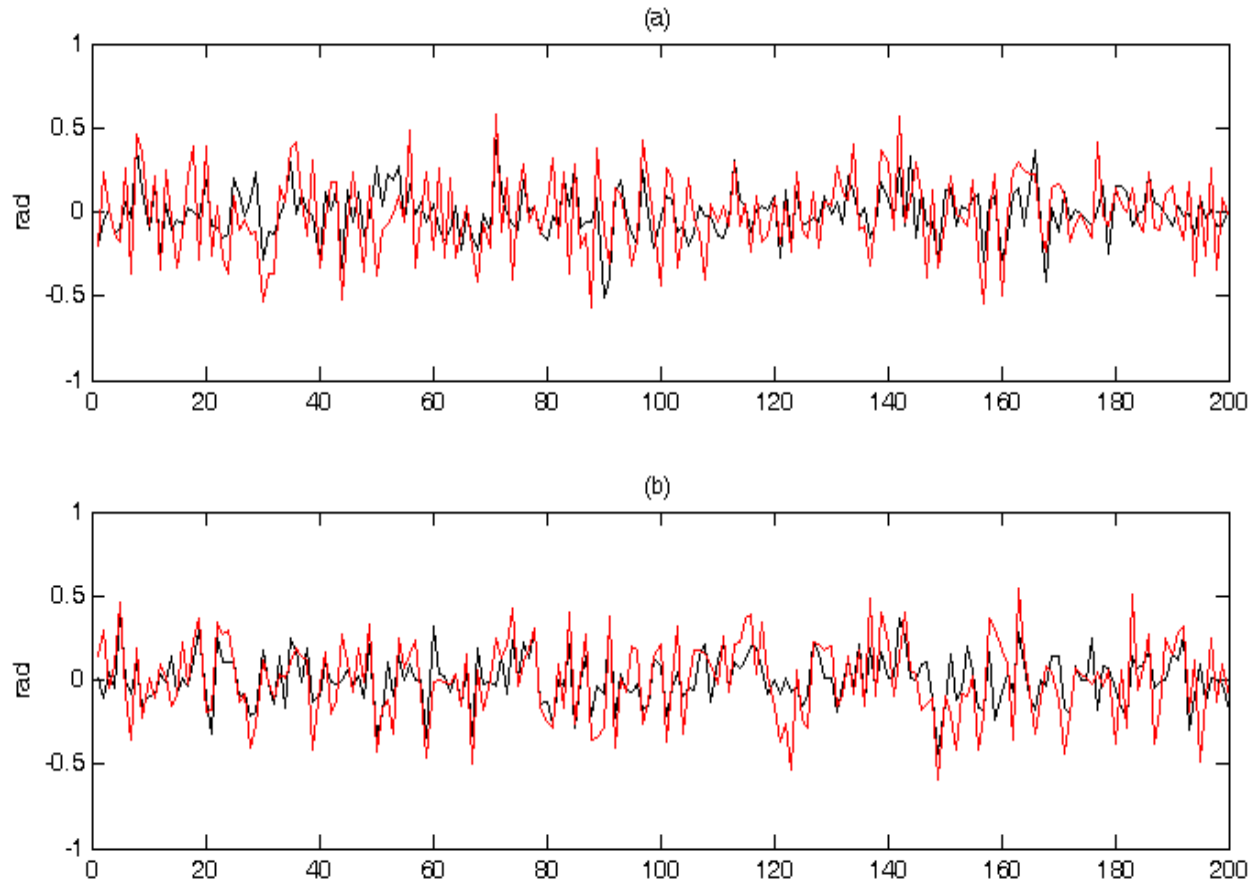
Defocus modes of Zernike aberration. Black curve show the calculated coefficients for phase screens which have 8 modes of Zernike aberrations and red curve show the calculated coefficient for original phase screen.

Astigmatisms



Astigmatism modes of Zernike aberration. Black curve show the calculated coefficients for phase screens which have 8 modes of Zernike aberrations and red curve show the calculated coefficient for original phase screen. (a) astigmatism 45 (b) astigmatism 0-90

Coma



Coma modes of Zernike aberration. Black curve show the calculated coefficients for phase screens which have 8 modes of Zernike aberrations and red curve show the calculated coefficient for original phase screen. (a) coma y (b) coma x .

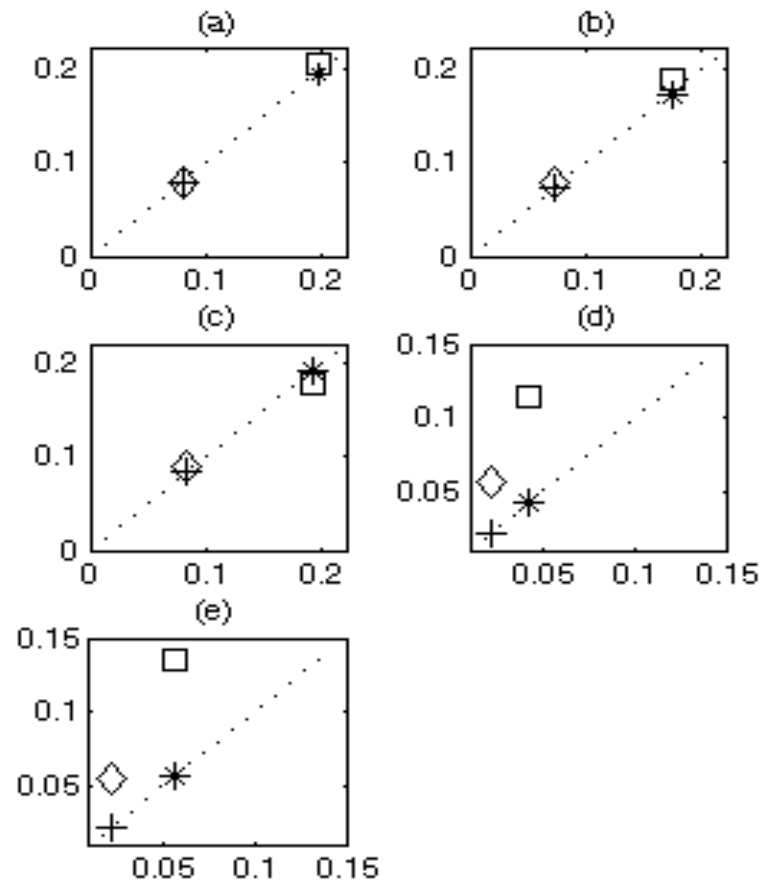
(a) Defocus

(b) Astigmatism 45

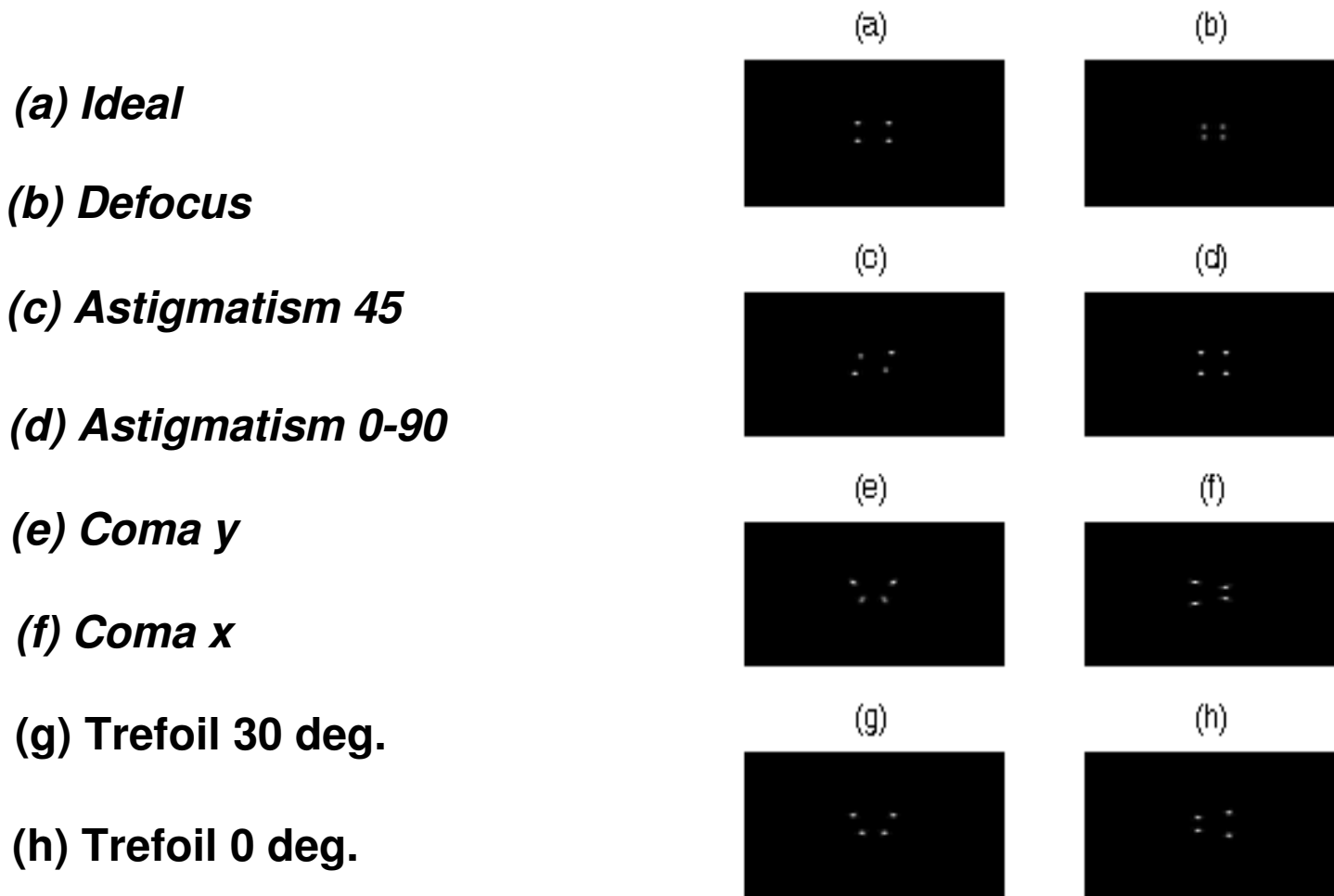
(c) Astigmatism 0-90

(d) Coma y

(e) Coma x



Horizontal axis is initial phasescreen variance (rad^2) and the vertical axis is the calculated variance by four apertures Hartmann (rad^2). Square and Diamond shows the original phase set. Star and Plus shows the set which each phase screen has 8 mode aberrations. (a) Defocus (b) Astigmatism ± 45 (c) Astigmatism 0-90 (d) Coma y (e) Coma x.



In figure we show the configuration of four spots for the defocus, the astigmatism, the coma and the trefoil terms. As shown in figure , the configuration of the four spots in presence of the trefoil aberration is the same as the configuration of four spots in presence of coma terms. This similarity makes error in measurements of coma aberrations coefficients.

The ideal four spots images

The ideal four spots images can be obtained by two methods:

1-Pointing the telescope on faint star and capture an image, the exposure time of this image must be long enough to average out turbulence effects but short enough to avoid any degradation that is due to telescope tracking errors.

2-Averaging on the centroid of large number of short exposure time images which captured for Fried parameters measurement to eliminate the seeing effects and finding the position of the centeroid of 4-spots images.

Conclusion

-we discuss the ability of Hartmann test with four apertures to measure the atmospheric primary aberrations both by the simulation procedure. We show by numerical simulations that this method is able to measure the defocus and astigmatism aberrations correctly.

-However, the results obtained for the coma aberration are not desirable.