# $Cn^2$ profile measurement from Shack-Hartmann data

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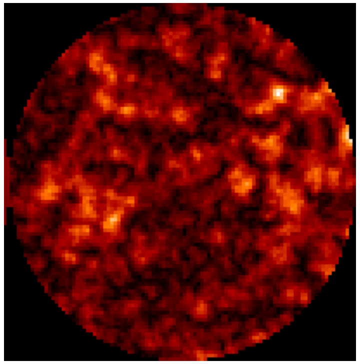
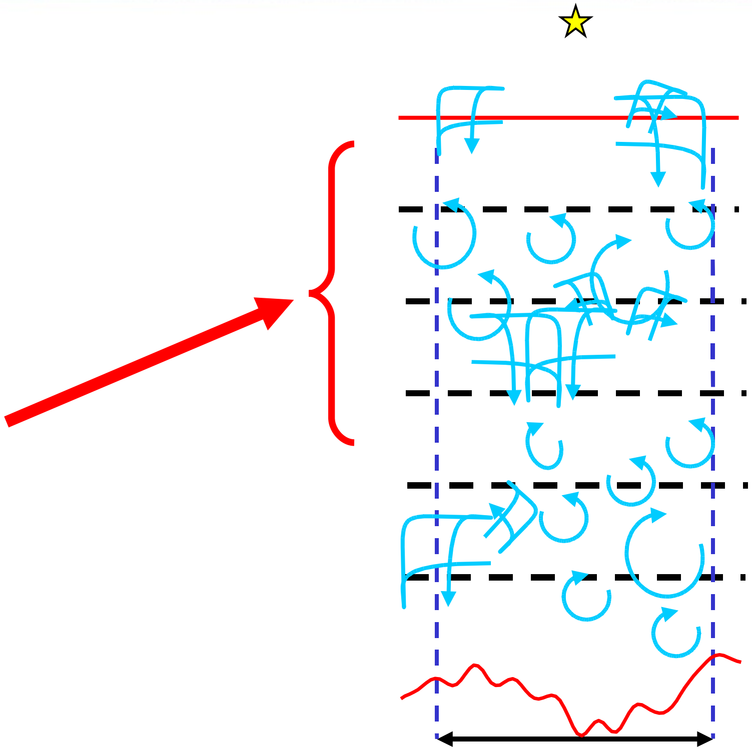
retour sur innovation

# A new method to profile $C_n^2$





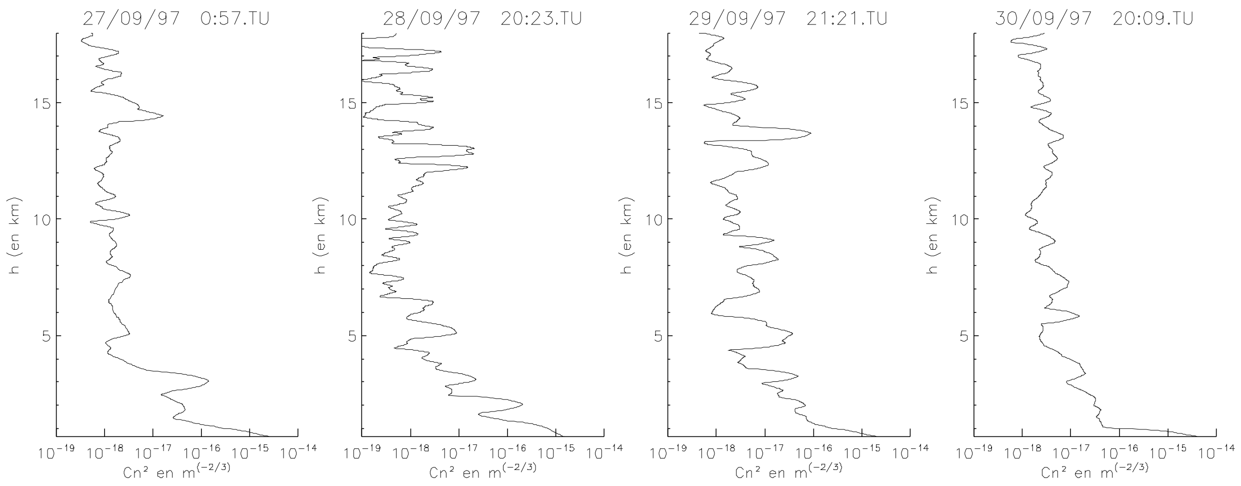

**Use of scintillation signal from Shack-Hartmann data**
**Coupled Slodar-Scidar (CO-SLIDAR) method**

# Outline

- Motivation and techniques
- Shack-Hartmann data: theoretical background
- Numerical validation to profile Cn2
- Experimental $Cn^2$ profiles

# Motivation for $C_n^2$ profile knowledge

- Dimensioning systems
- Evaluation of performances
- A priori for servo-loop laws



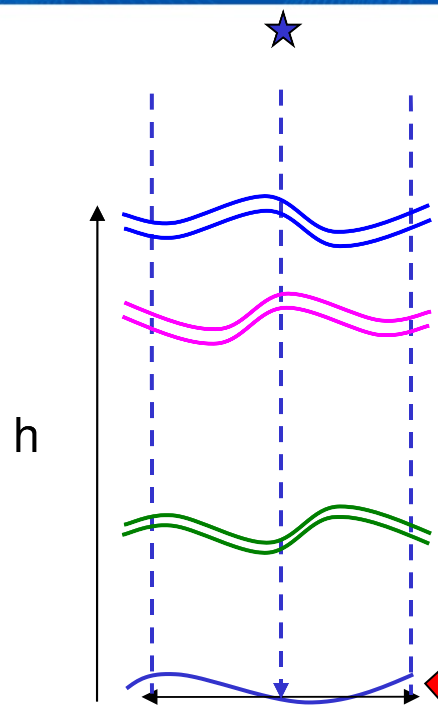

Profile from Observatoire de Haute Provence (balloon probes)

High variability

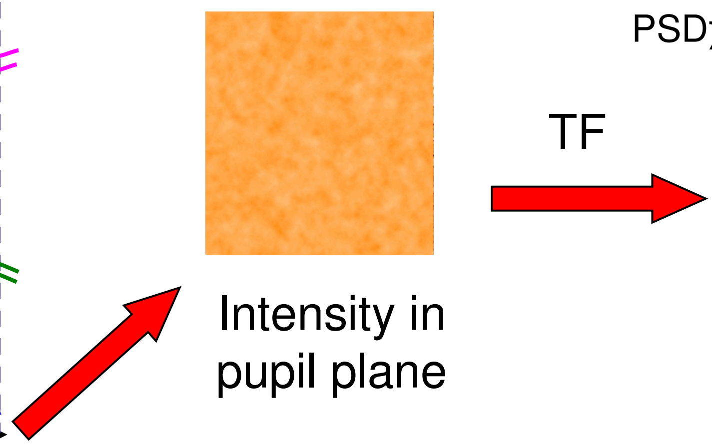
Need of profile monitoring

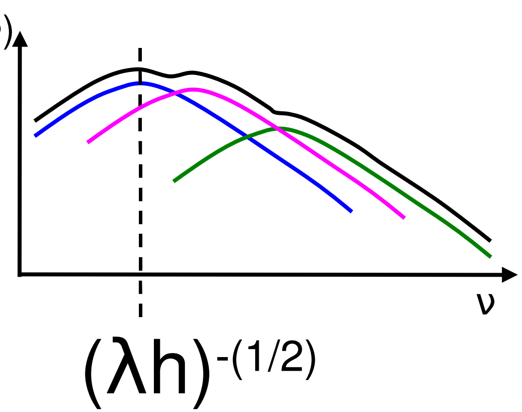
**How to measure $Cn^2$ profile ? by optical means**

# Methods of $C_n^2$ profiling: single source




Spectral analysis of scintillation structures



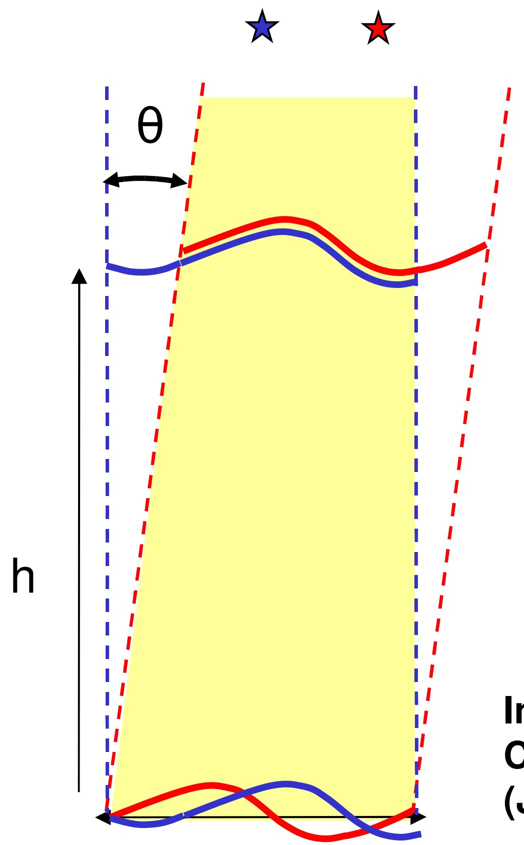



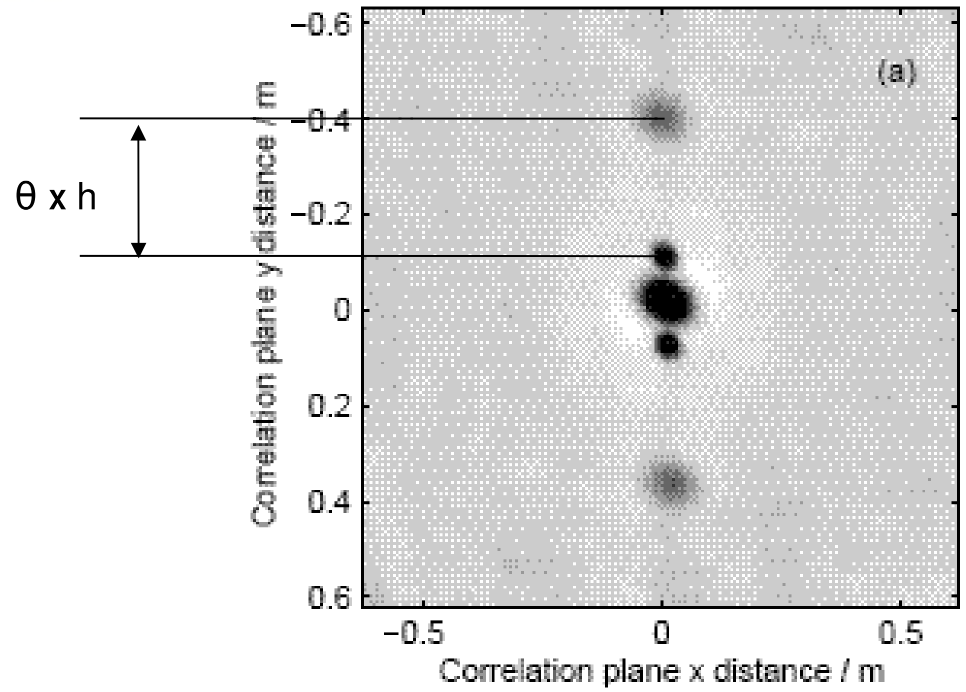

No sensitivity to law altitude layers (no propagation) → More operations needed (mode: « generalized») → **More uncertainties**

MASS (V. Kornilov, A. Tokovinin) SSCIDAR (D. Garnier)

**Single source: medium-low vertical resolution**

# Methods of $C_n^2$ profiling: multiple sources




**Intensities:**
**Cross-correlations of scintillation indices: G-SCIDAR (J. Vernin, V.A. Klueckers)**

**Slopes:**
**Cross-correlations of wavefront slopes: SLODAR (R.W. Wilson)**

**What about simultaneous exploitation of slopes and intensities ?**

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- **Shack-Hartmann data: theoretical background**
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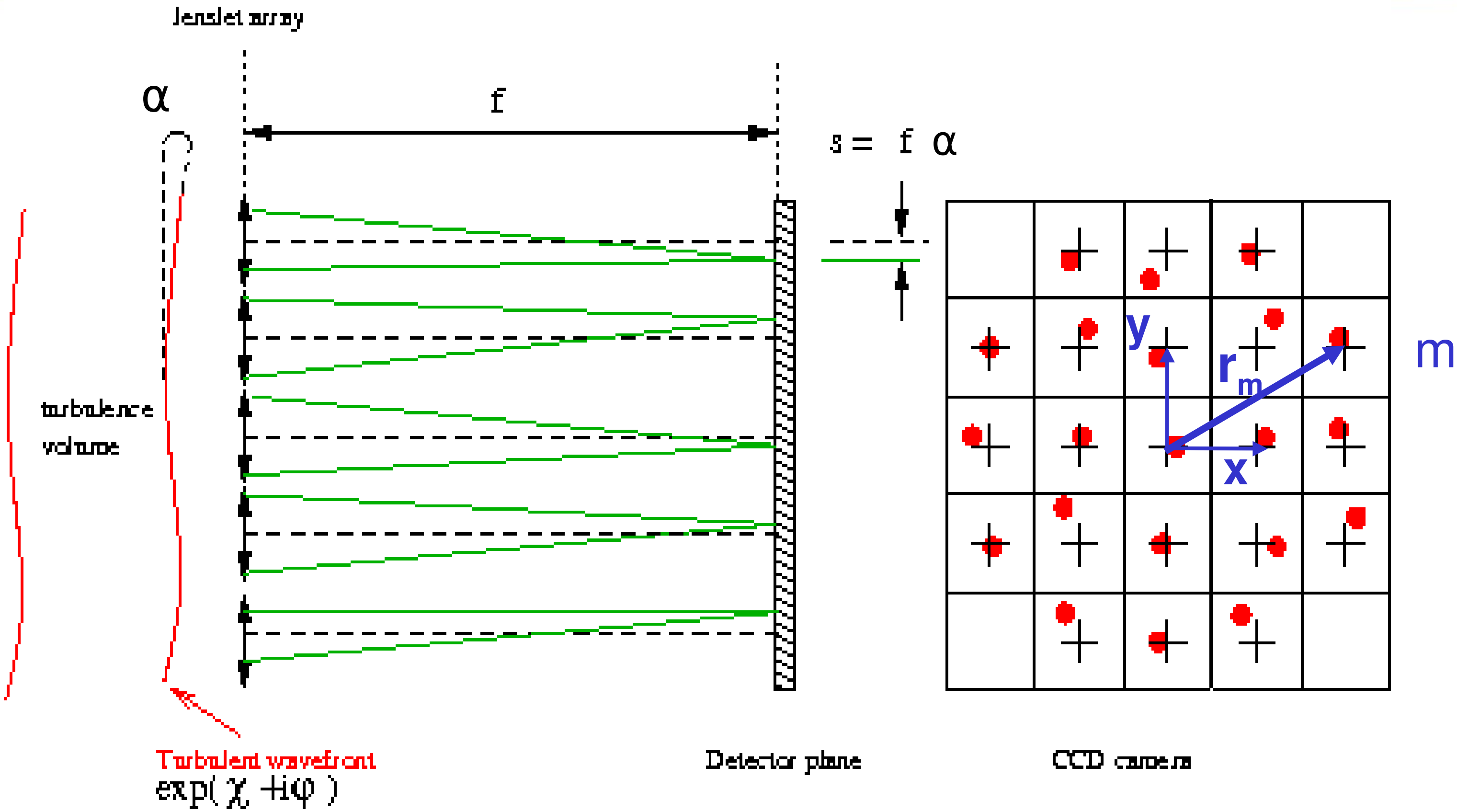
# Shack Hartmann Wavefront sensor

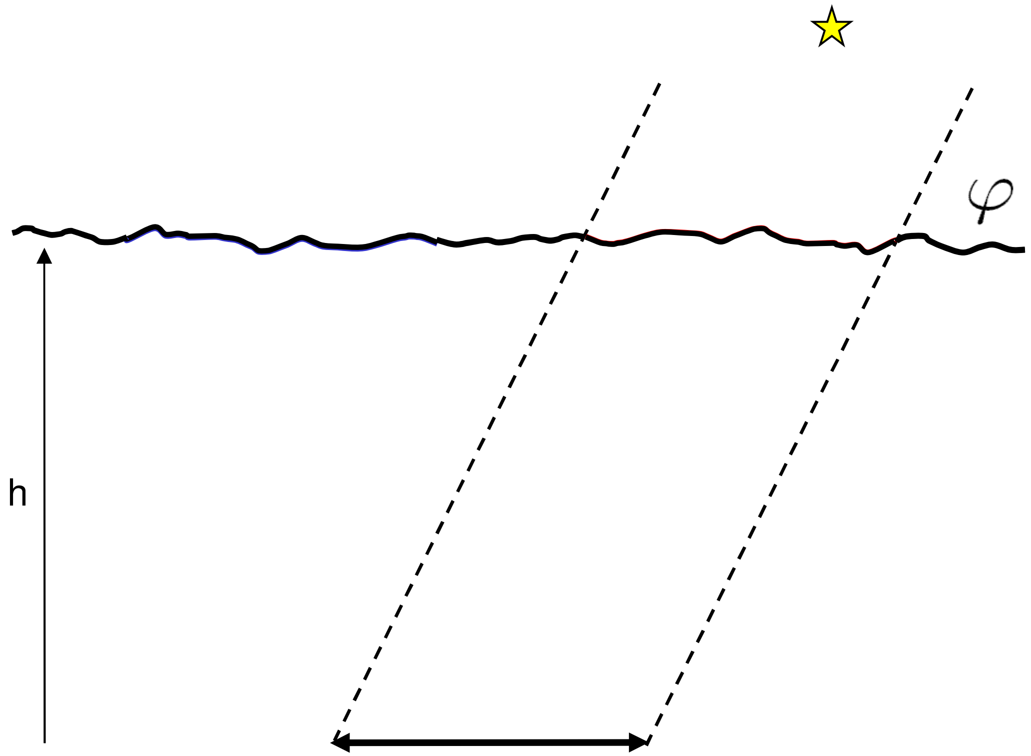



SH data:

$\mathbf{s_m(\theta)}$ = wavefront slopes averaged on subaperture at $\mathbf{r_m}$

$i_m(\boldsymbol{\theta})$ = averaged intensity of the incident wave on subaperture at $\mathbf{r_m}$

# Theoretical background



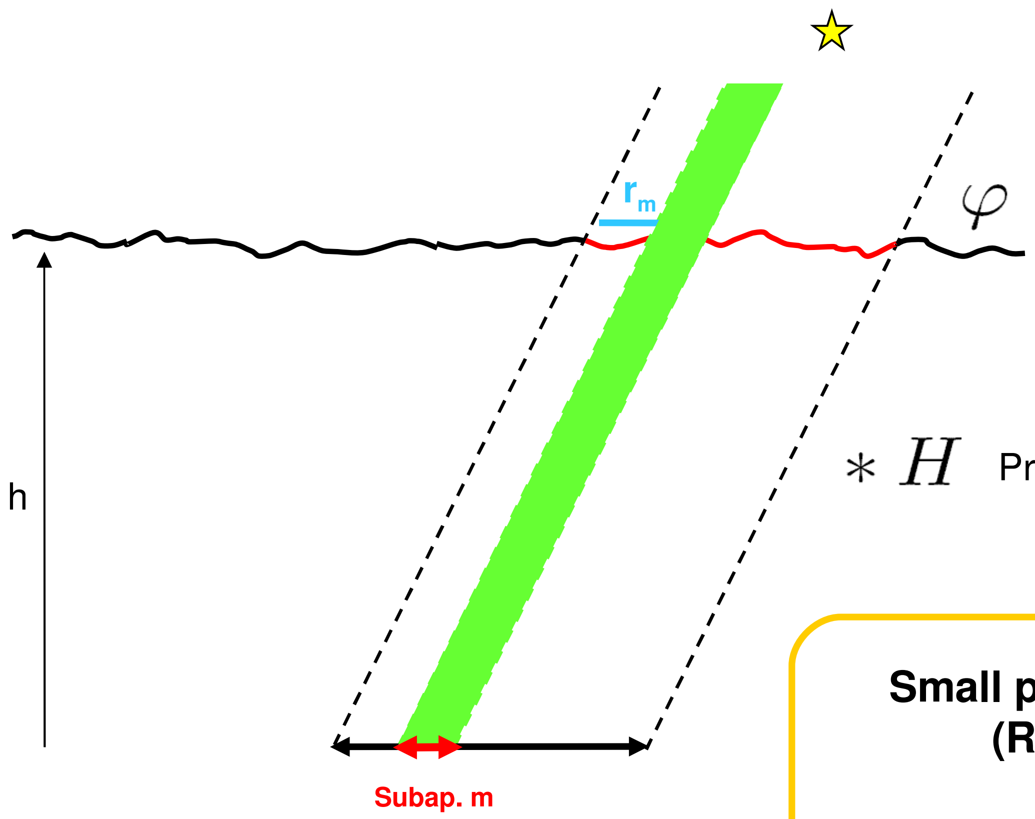

Turbulent volume : N layers

Small perturbations layers are independant

1 layer at altitude h

Phase perturbation $\phi$

# Theoretical background

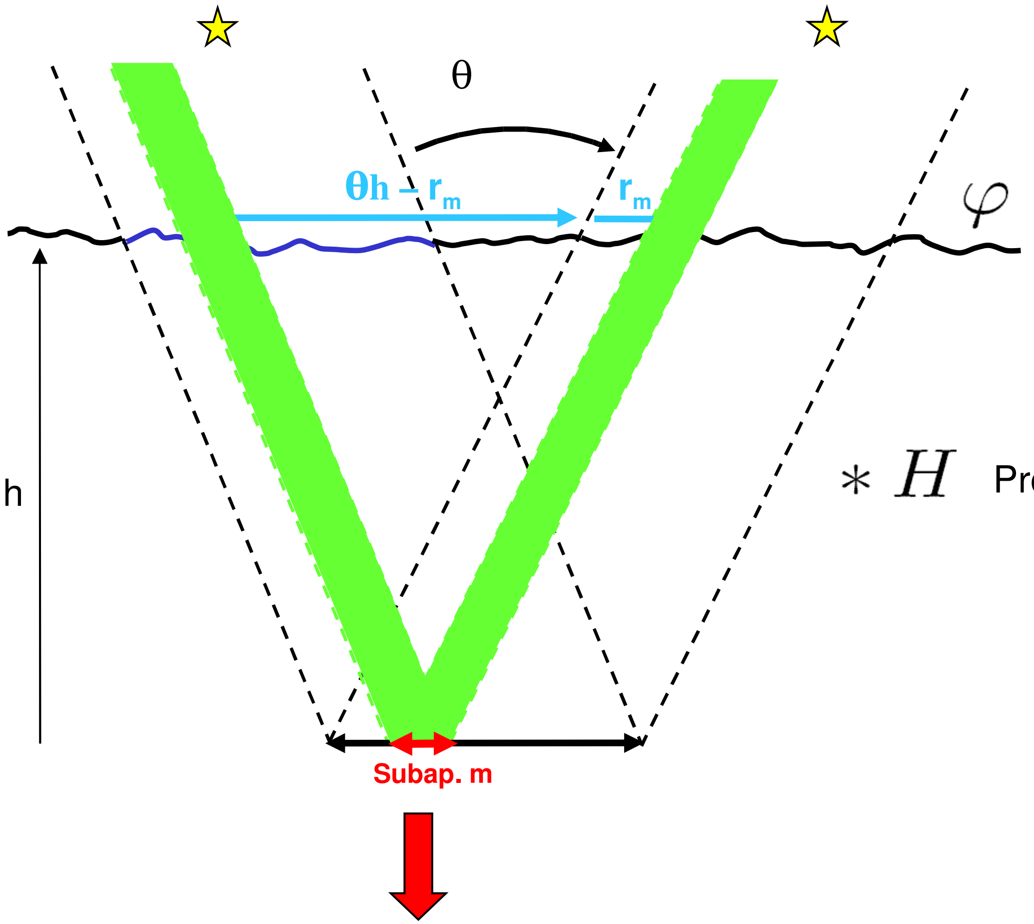



$* H$ Propagation + subaperture averaging

**Small perturbation approximation**
**(Rytov regime, $\sigma_\chi^2 < 0.3$)**

$$s^x_m(\mathbf{0}) = \varphi * H * \delta(\boldsymbol{r} - \boldsymbol{r}_m)$$

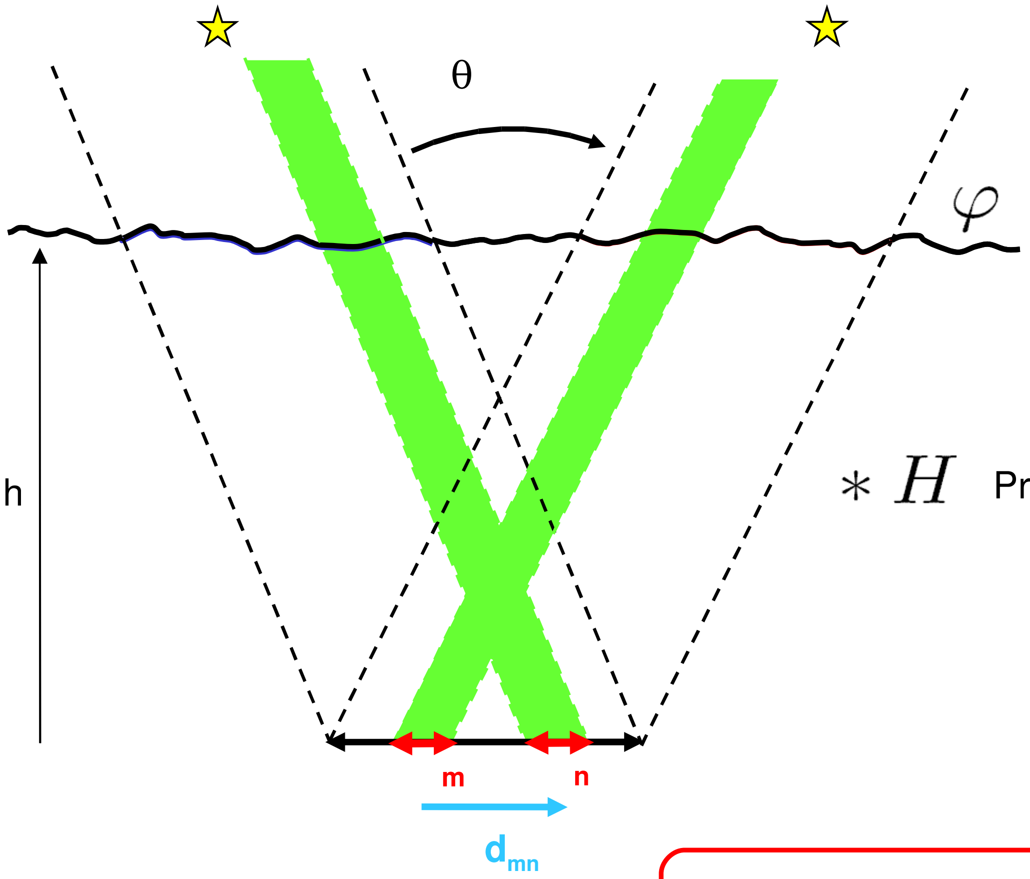
# Theoretical background




$* H$ Propagation + subaperture averaging

**Correlations slopes & of intensities**

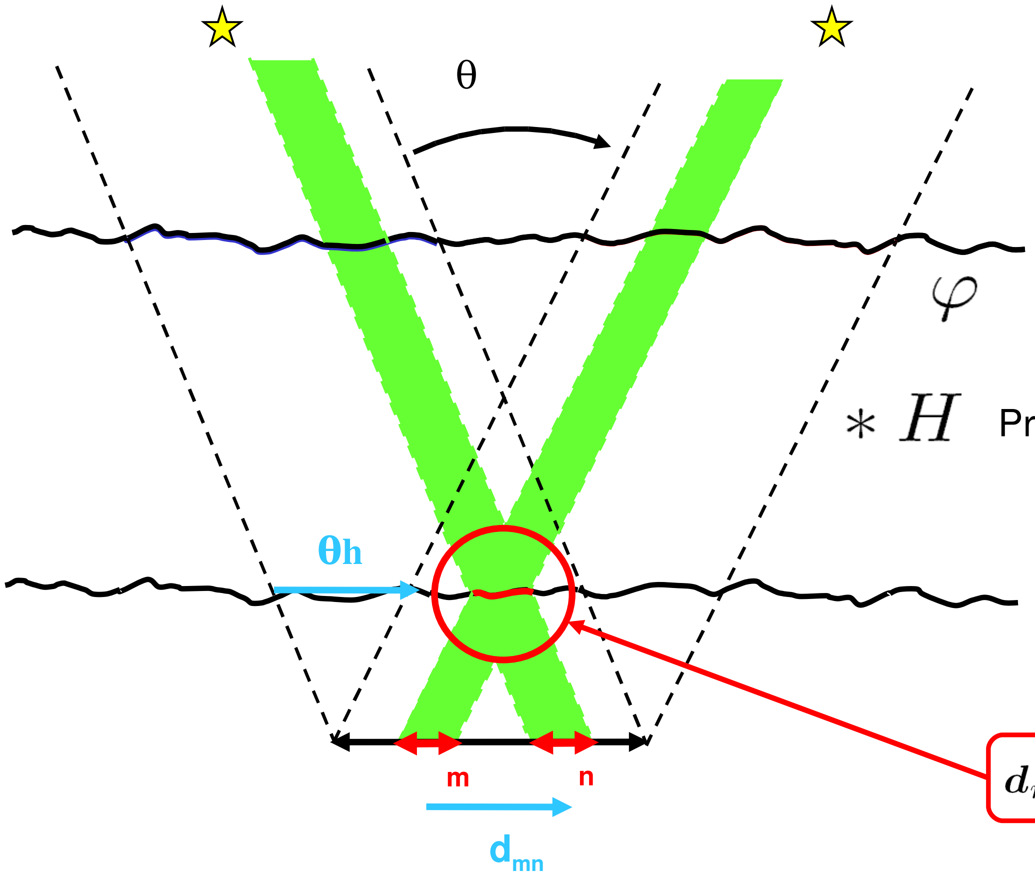
# Correlations of slopes




$* \, H$ Propagation + subaperture averaging

$$\langle s_m^x(\mathbf{0}) s_n^x(\boldsymbol{\theta}) \rangle = C_n^2(h) W_{ss}^{xx}(h, \mathbf{d}_{mn}, \boldsymbol{\theta}) dh$$

# Correlations of slopes (& intensities)




$\varphi$

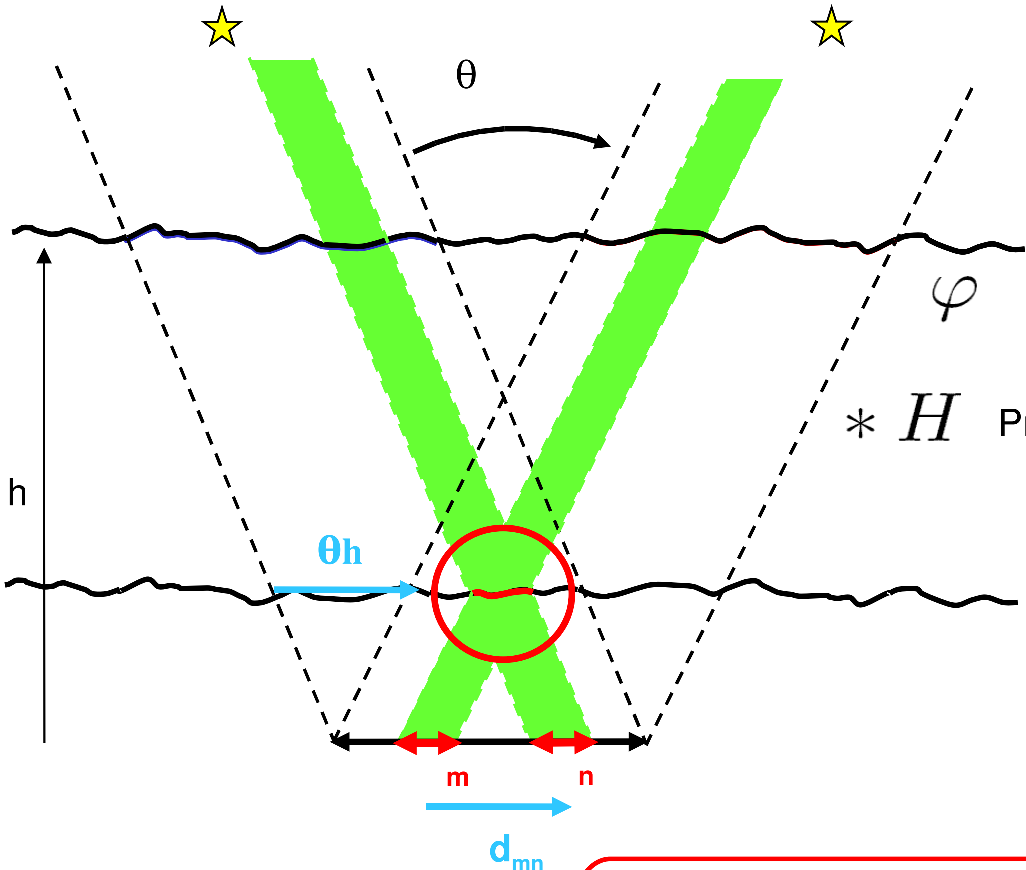
$* H$ Propagation + subaperture averaging

Altitude of maximum sensitivity

$$\boldsymbol{d}_{mn} = \boldsymbol{\theta} h$$

$$\langle s_m^x(\mathbf{0}) s_n^x(\boldsymbol{\theta})\rangle = C_n^2(h) W_{ss}^{xx}(h, \boldsymbol{d}_{mn}, \boldsymbol{\theta}) dh$$

# Correlations of slopes(& intensities)




$* H$ Propagation + subaperture averaging

$$\langle s_m^k(\mathbf{0}) s_n^l(\boldsymbol{\theta})\rangle = \int_0^\infty C_n^2(h) W_{ss}^{kl}(h, \mathbf{d}_{mn}, \boldsymbol{\theta}) dh$$

*Measurement* *unknown* *Weighting*

# Shack-Hartmanndata: correlations

| | | | | |
|---|---|---|---|---|
| **Slopes** | $\langle s_m^k(\mathbf{0}) s_n^l(\boldsymbol{\theta})\rangle$ | ⟷ | SLODAR | Shack-Hartmann ++ !! |
| **Intensities** | $\langle i_m(\mathbf{0}) i_n(\boldsymbol{\theta})\rangle$ | ⟷ | SCIDAR, MASS | |
| **Coupling** | $\langle s_m^k(\mathbf{0}) i_n(\boldsymbol{\theta})\rangle$ | ⟷ | $\phi$ | |

Vector of covariance measurements estimated from a finite number of recorded frames



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# Problem statement

SH data: **$s_m(\theta)$**, $i_m(\theta)$: $x_k^i$

**Estimated covariances:**

$$\overline{X^i X^j} = \frac{1}{N-1}\sum_{k=1}^{N}(x_k^i - \mu^i)(x_k^j - \mu^j)$$

**Vector data**

$$C_{mes} = \begin{pmatrix} \overline{s_m^k(\mathbf{0})s_n^l(\mathbf{0})} \\ \overline{i_m(\mathbf{0})i_n(\mathbf{0})} \\ \overline{s_m^k(\mathbf{0})i_n(\mathbf{0})} \end{pmatrix}$$

ou

$$C_{mes} = \begin{pmatrix} \overline{s_m^k(\mathbf{0})s_n^l(\boldsymbol{\theta})} \\ \overline{i_m(\mathbf{0})i_n(\boldsymbol{\theta})} \\ \overline{s_m^k(\mathbf{0})i_n(\boldsymbol{\theta})} \\ \overline{s_m^k(\mathbf{0})s_n^l(\mathbf{0})} \\ \overline{i_m(\mathbf{0})i_n(\mathbf{0})} \\ \overline{s_m^k(\mathbf{0})i_n(\mathbf{0})} \end{pmatrix}$$

**Single source** **Multiple sources**

Direct problem: $C_{mes} = \mathcal{M}C_n^2$

$\mathcal{M}$ : weighting functions

$C_d$ : covariance of detection noise (bias)

$n$ : statistical noise on $C_{mes}$

# Inversion of direct problem

## Vector data

Calibration → Subtraction of detection noise bias

$$\hat{C}_{mes} = C_{mes} - C_d$$

## Noise treatment

Limited statistic (convergence noise) → Covariance matrix $C_{noise}$

## Criterion to minimise relatively to S ($C_n^2$ profile)

$$J = \underbrace{(\hat{C}_{mes} - MS)^t C_{noise}^{-1} (\hat{C}_{mes} - MS)}_{\textbf{Data likelihood}} + \underbrace{\beta \|\Delta\ S\|^2}_{\textbf{A priori}}$$

$\beta$ : regularisation parameter

**Minimisation of J with positivity constraint**

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# Simulation

Object model: binary star

**θ = 10 arcsec**

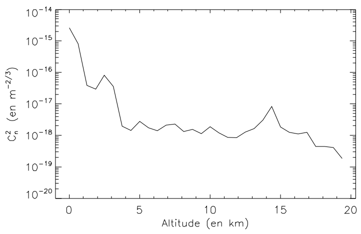
+

Code PILOT

Simulation of turbulent screens

+

Diffractive propagation




**32 layers/ 400 frames**

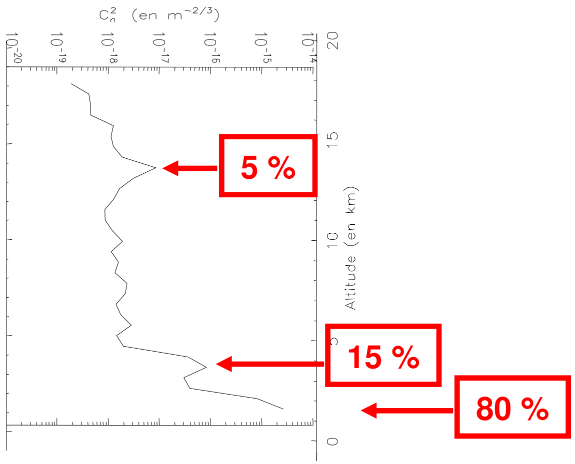
+

Shack-Hartmann: 16 x 16, d = 2.5 cm, λ = 0.5 µm (D = 40 cm)

**Data**: $\mathbf{s_m(\theta)}$, $i_m(\theta)$ → $\hat{C}_{mes}$

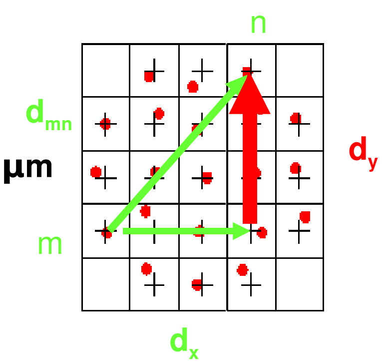
# Complementarities of slopes & intensities

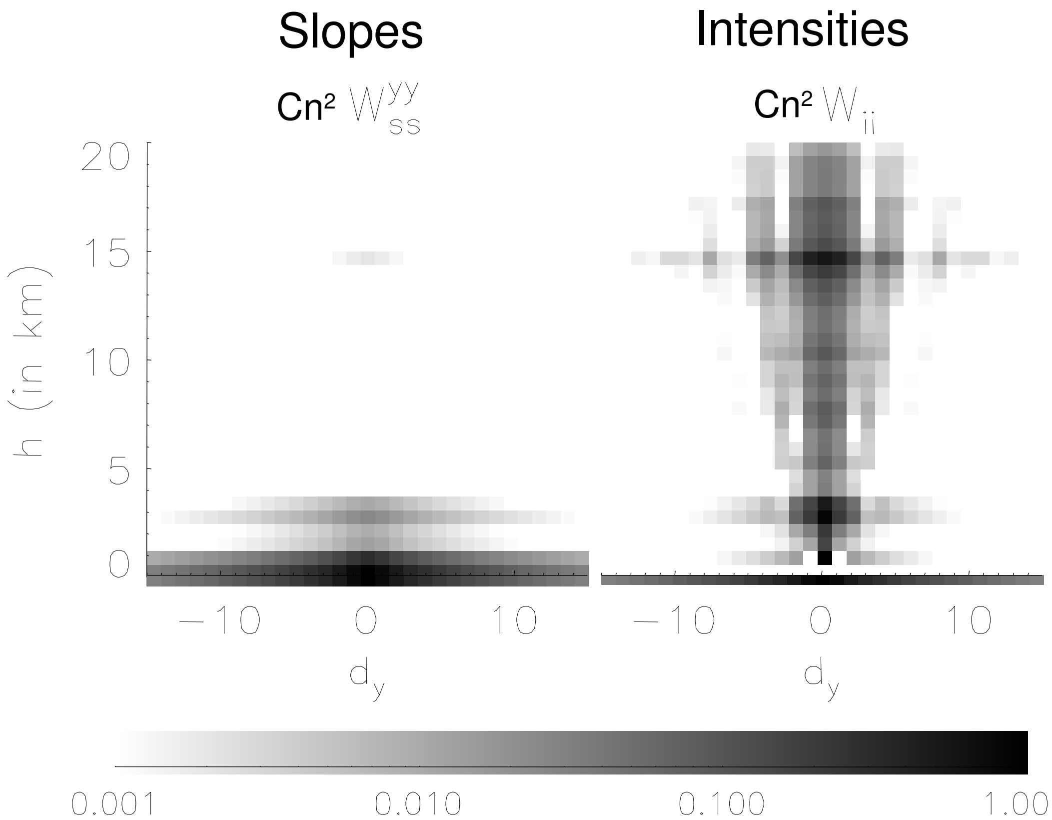
Cn²




**Shack-Hartmann**

**D= 0.4 m,16 x 16, λ = 0.5 µm**




**Simultaneous exploitation: better sensitivity**



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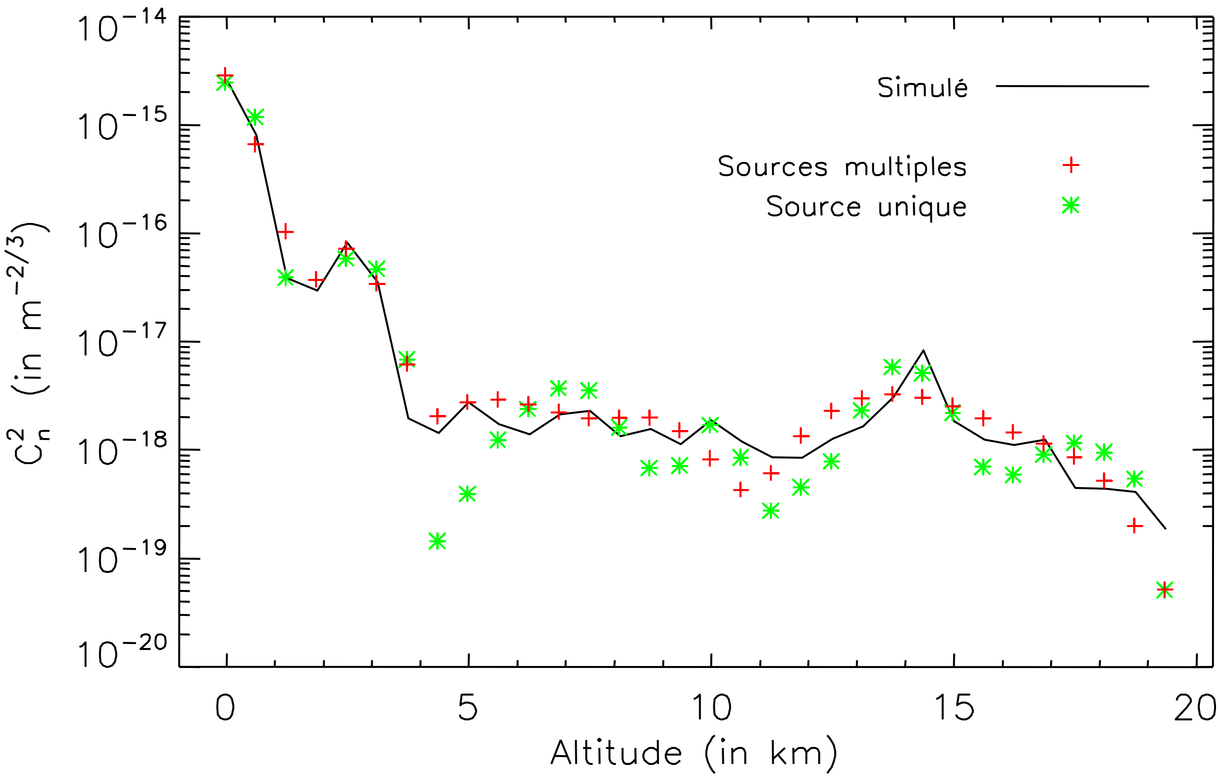
# Cn² estimation: results in simulation




N. Védrenne, V. Michau, C. Robert, J.-M Conan, « Full exploitation of Shack-Hartmann data for $C_n^2$ profile measurement », OL, octobre 2007

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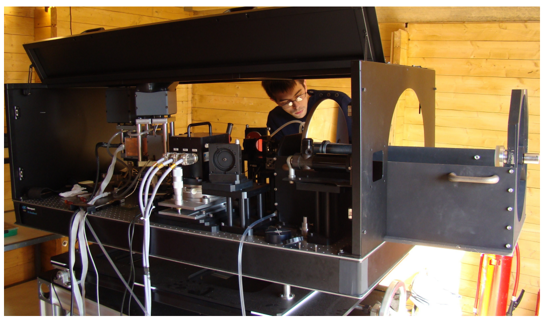
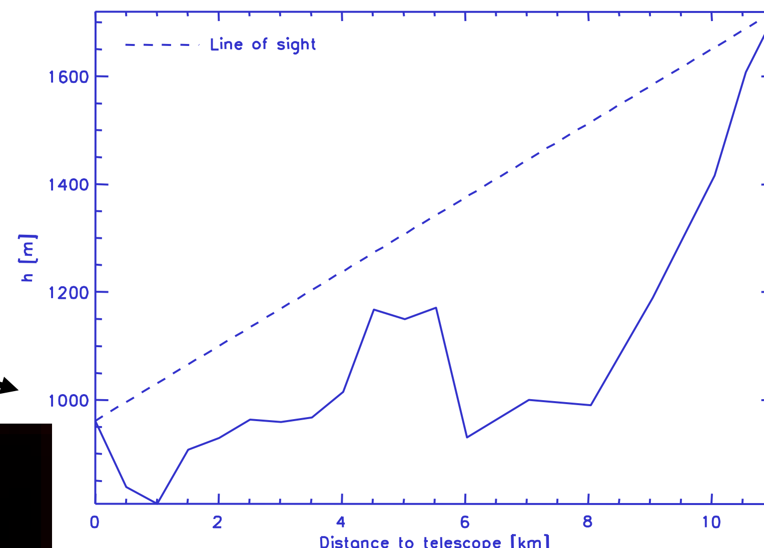
# AO on extended sources in the infrared


**Telescope diameter : 350 mm**

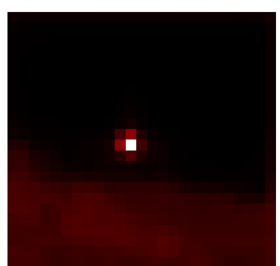


**Projector position**




**Observation site**

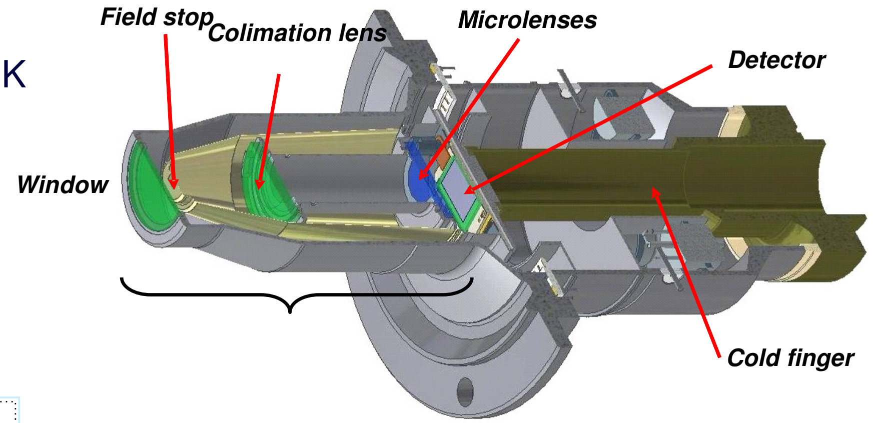
**For profile measurements:**
**halogen projector (high flux source)**

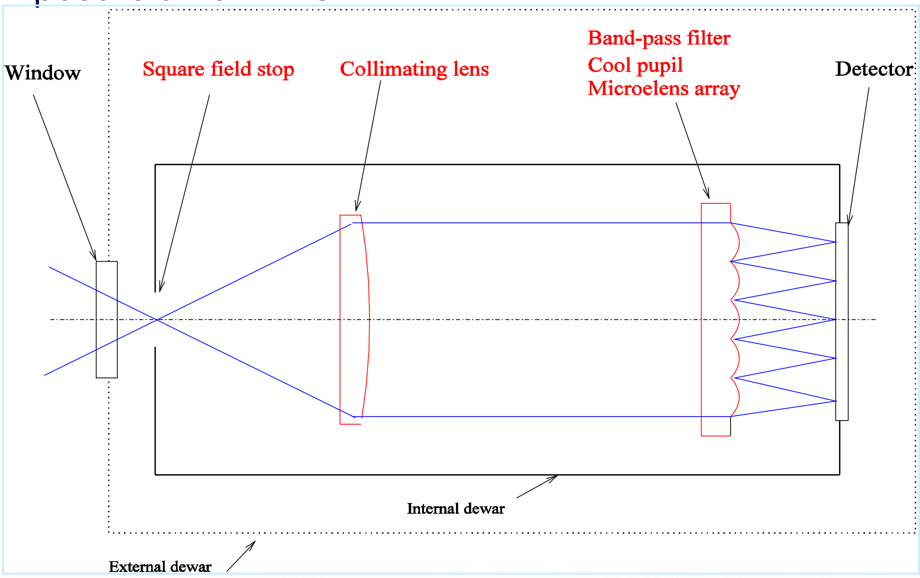


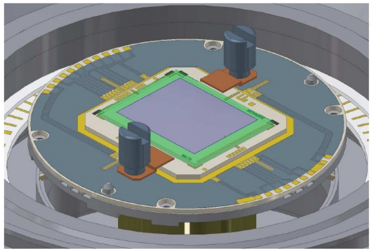
# Shack-Hartmann Wavefront Sensor in IR

## SOFRADIR/ONERA (Robert et al, SPIE 2007)

- 5x5 sub-apertures
- 3.4- 4.2 µm spectral bandwidth (MCT), cryostat T°: 90 K
- FOV: 14 m @ 11 km
- Shannon/2 sampling @ 3.7 µm
- 125 x 125 pixels, 30 µm pixels
- Well capacity: 1,4 Me
- Quantization: 14 bits
- Calibration : conversion factor (1 ADU -> phe: 133)
- Detector Noise: 200 phe
- Exposure time: 2 ms



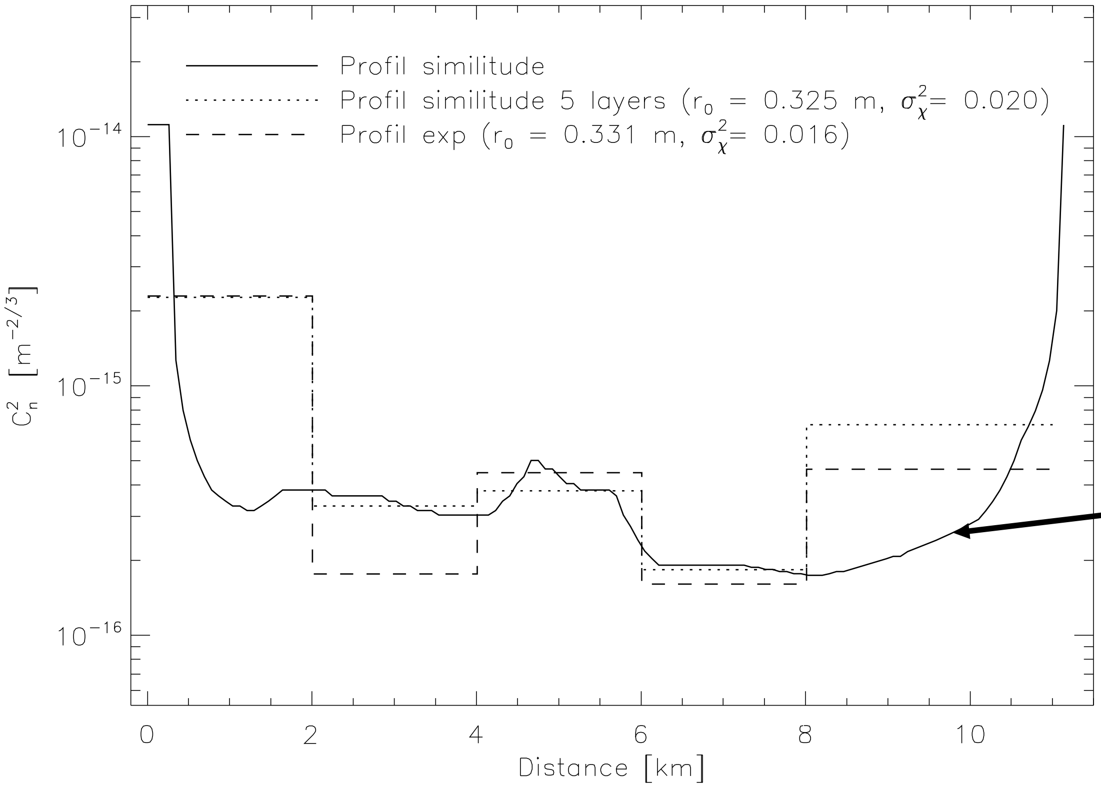



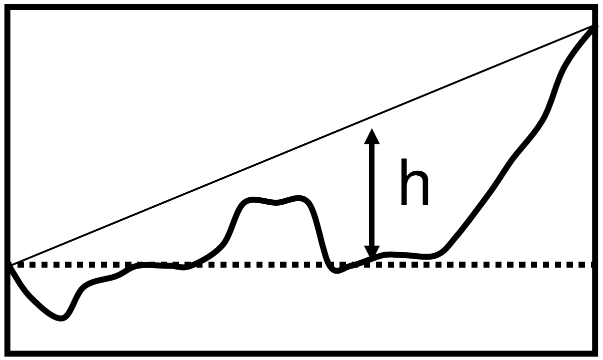
HgCdTe CMOS

# Cn2 profile with SCO-SLIDAR: 1 source

**Nightfall, ground T° < air T° ~ stable atmosphere**

**Theoretical profile : similitude profile in $h^{-2/3}$**




$\alpha\ h^{-2/3}$

Concordance between similitude profile and average profile

*N. Védrenne, et al. SPIE octobre 2010*

# On the way to profile Cn2 with CO-SLIDAR: 2 sources

**Master 2 of Juliette Voyez**

- Need of position and flux estimation in SH subimages reconstarfield algorithm: ⇨
  - Accurate slopes and intensities of stars
  - Qualification in simulation
- Processing of SH images from astronomical data (IAC)
- Direct model for wind profile in the CO-SLIDAR formalism

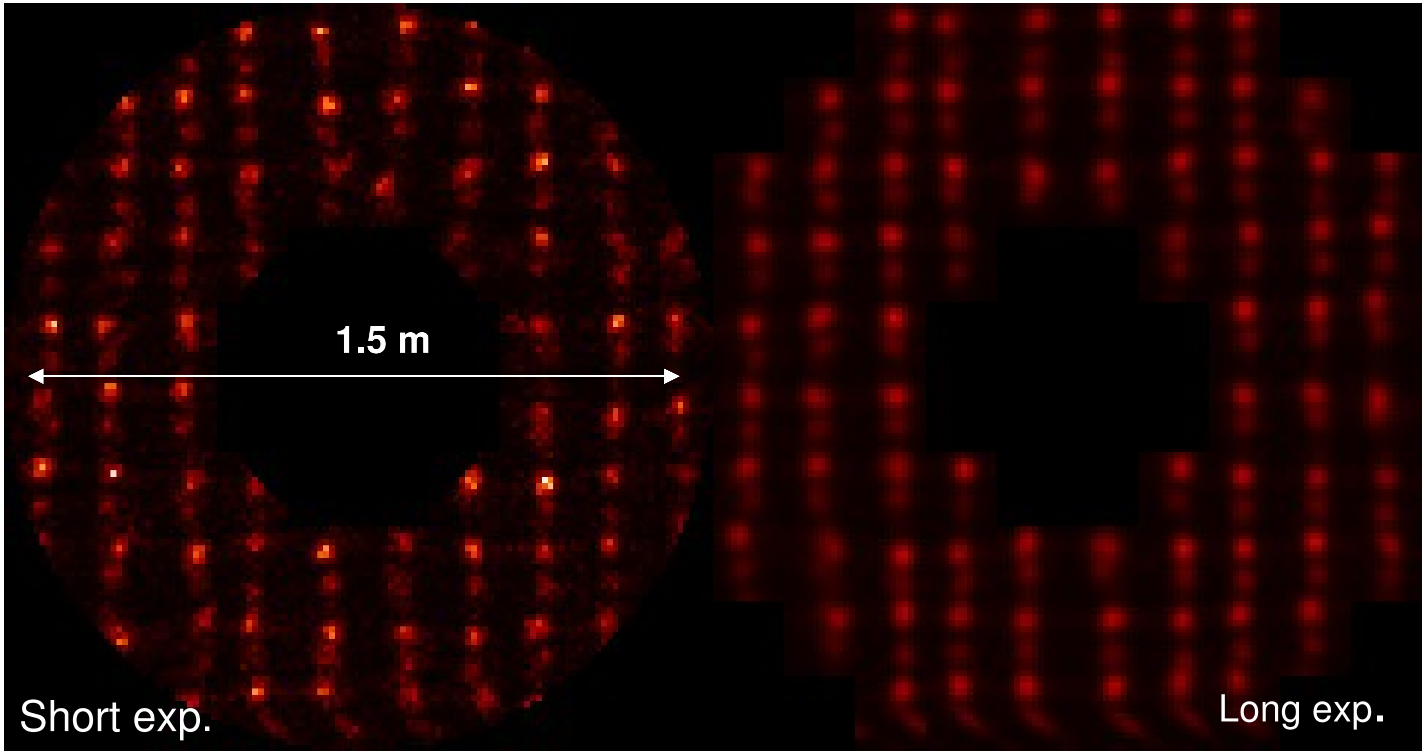
# Binary star on SH images from IAC

- Telescope Carlos Sanchez at Teide
  - Cassegrain
  - Diameter D = 1,5 m
  - Focal dist.  21 m
  - Aperure number F/13,8

- SH-WFS
  - 10x10 microlens array
  - Diameter  d = 15 cm
  - Focal  34 mm
  - Pitch 312,5 µm

- Detector CCD
  - 120x120 pixels$^2$
  - $\lambda$ = 0,6 µm
- Subimages
  - 12x12 pixels$^2$

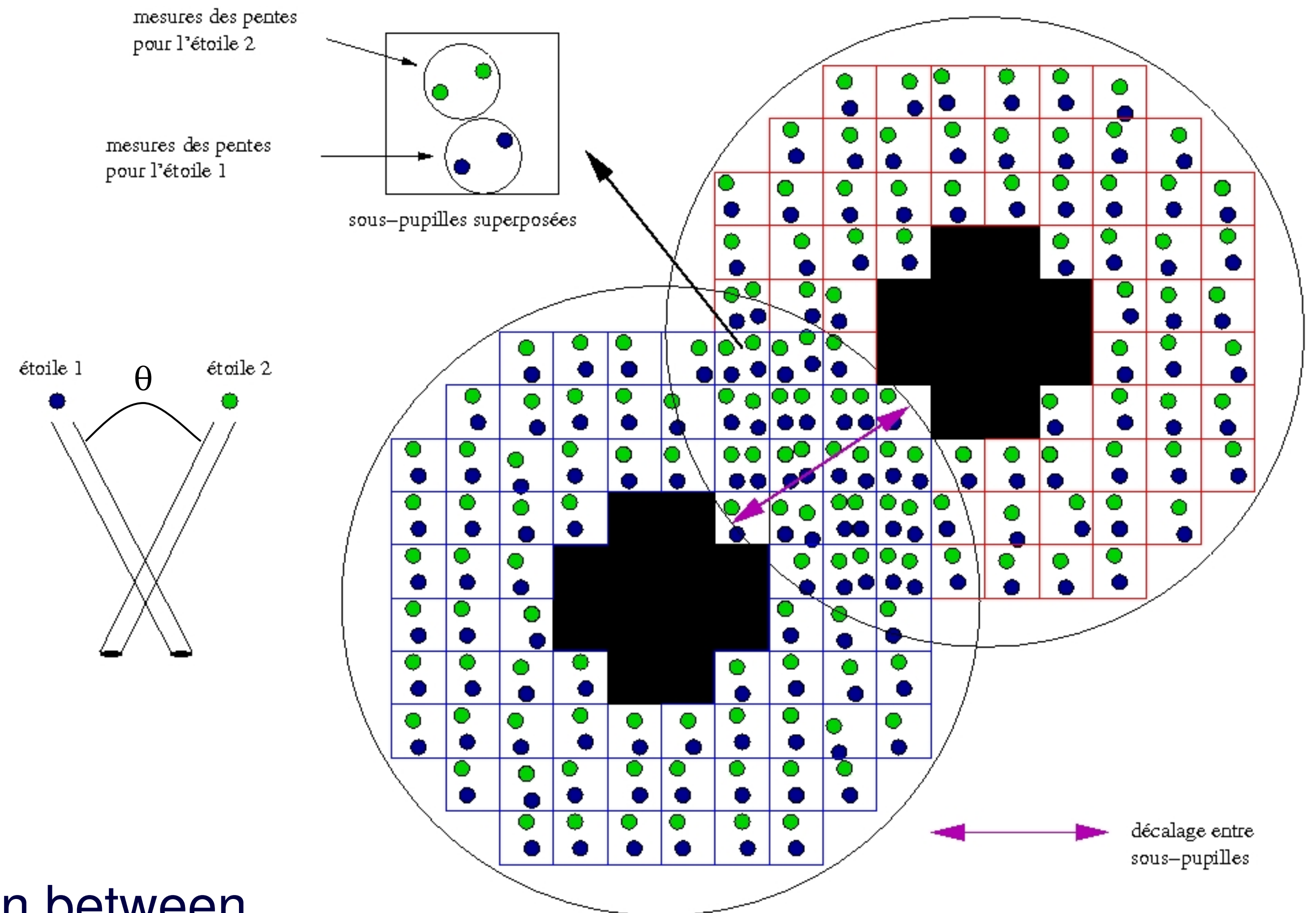
5000




**Star separation 5.6 "**

# Making cross-correlations

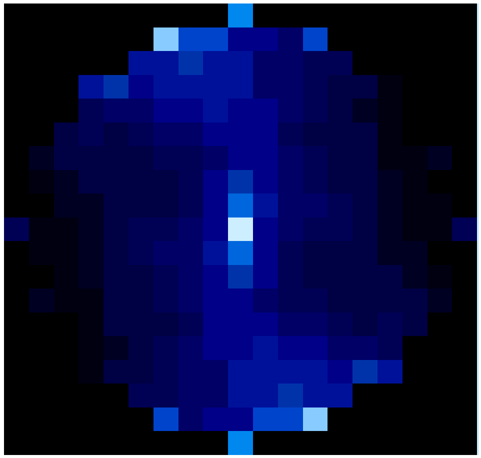
auto/inter correlations maps




shows the correlation between all couples of subap. having the same gap

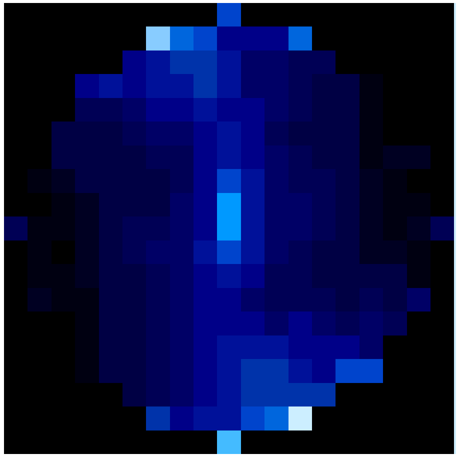
$C_n^2$ profile

# Reconstruction of IAC $Cn^2$ profile
## with slopes only (SLODAR)

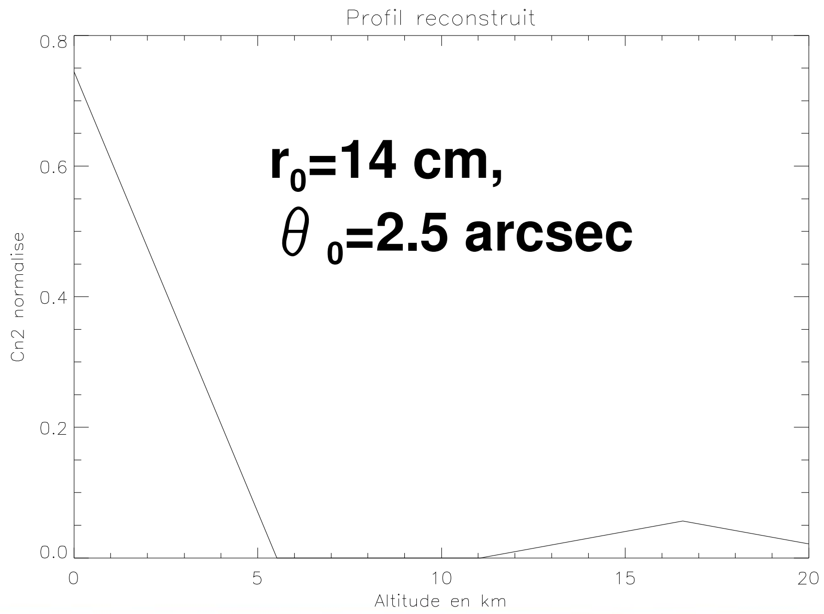


**Auto-correlation**

**1D deconvolution**
**Validated with turbulence simulator on the MCAO bench at ONERA**



**Inter-correlation**



# Conclusion and perspectives

**Proposition of two original methods to profile $C_n^2$**

New exploitation of the Shack-Hartmann

Sensitivity

Validated numerically

**PhD**

**Processing of real data (SCO-SLIDAR, SLODAR, CO-SLIDAR)**

Study of noise effect (photons, detector, quantification)

Calibration

Adaptation to close binary, moon edge, sun edge

Determination of wind profile

Influence of external scale?